The Non-Existence of Certain Laguerre-Chebyshev Quadrature Formulas

By F. D. Burgoyne

Salzer [1] considers quadrature formulae of the form

$$\int_a^\infty w(x)f(x)\,dx = W\sum_{i=1}^{n} f(a_i),$$

where in the first case $a = 0$ and $w(x) = e^{-x}$, and in the second case $a = -\infty$ and $w(x) = e^{-x^2}$. In particular, he shows that in the first case the a_i are not all real and non-negative for $n = 3(1)10$, and in the second case the a_i are not all real for $n = 4(1)10$.

The object of this note is to summarise an extension of these results which has been carried out on Pegasus using one of Salzer's methods. As he shows, the a_i in the first case are the roots of

$$\sum_{j=0}^{n} A_j x^j = 0,$$

where

$$A_0 = 1$$

and

$$A_j = \frac{-n}{j}\sum_{k=1}^{j} k!\,A_{j-k}$$

for $j = 1(1)n$. Hence, if s_n denotes the number of sign changes in the sequence $A_0, A_1, \cdots, A_n$ we may employ Descartes' rule of signs to infer that the maximum number of real non-negative a_i is s_n. Similarly, in the second case the a_i are related to the roots of

$$\sum_{j=0}^{[n/2]} A_j x^j = 0,$$

where

$$A_0 = 1$$

and

$$A_j = \frac{-n}{2j}\sum_{k=1}^{j} (k - \tfrac{1}{2})\,(k - \tfrac{3}{2})\cdots\tfrac{1}{2}\,A_{j-k}$$

for $j = 1(1)[n/2]$. If s_n now denotes the number of sign changes in the sequence $A_0, A_1, \cdots, A_{[n/2]}$, the maximum number of real a_i is $2s_n$ or $2s_n + 1$, according as n is even or odd. Proceeding on these lines, the following tabulation was obtained, which represents an extension of the results of the first paragraph to $n = 50$.

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n	First Case: Maximum Number of Real Non-Negative Abscissae	Second Case: Maximum Number of Real Abscissae	n	First Case: Maximum Number of Real Non-Negative Abscissae	Second Case: Maximum Number of Real Abscissae
11	5	7	31	11	11
12	7	6	32	11	10
13	7	7	33	11	11
14	7	6	34	13	10
15	7	7	35	13	11
16	7	6	36	13	10
17	7	7	37	13	11
18	9	6	38	13	14
19	9	7	39	13	15
20	9	10	40	13	14
21	9	11	41	13	15
22	9	10	42	13	14
23	9	11	43	13	15
24	9	10	44	15	14
25	9	11	45	15	15
26	11	10	46	15	14
27	11	11	47	15	15
28	11	10	48	15	14
29	11	11	49	15	15
30	11	10	50	15	14

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1. H. E. Salzer, "Equally weighted quadrature formulas over semi-infinite and infinite intervals," *J. Math. Phys.*, v. 34, 1955, p. 54–63.

Search For Largest Polyhedra

By Donald W. Grace

The configuration of eight points on the unit sphere which determines the convex polyhedron of maximum volume is not known. It is not the cube, whose volume is $8\sqrt{3}/9$, or about 1.5396, since the volume of the double pyramid (one point at each of the poles and the other six distributed uniformly around the equator) is $\sqrt{3}$, or about 1.732. [1, page 7]. The purpose of the present work was to determine if an even larger polyhedron could be found.

The search was carried out using gradient methods on the Burroughs 220 computer. Each set of eight points on the sphere may be interpreted as a vector, W, in Euclidean 16-space. For each W, let $V(W)$ denote the volume of the convex hull of the eight points, and $G(W)$ the gradient of $V(W)$. A starting configuration was introduced into the computer and modified iteratively. At each iteration, the 16-dimensional vector, W, was replaced by $W' = W + M \cdot G(W)$, for some real M

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